[Mock Test 31 questions]

1. (Integer) If *a*, *b*, and *c* are integers, 33 = *abc*, and *a* < *b* < *c*, what is the smallest possible value of *a*?

A. -33 B. -3 C. 3 D. 11 E. 33

2. (Inequality) If *a* is an integer greater than 4, but less than 21 and *b* is an integer greater than 14 but less than 31, what is the range of $\frac{a}{b}$?

A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{5}{6}$ D. 1 E. $\frac{7}{6}$

3. (Ratio) All data in a certain school is recorded either on the computer or on paper. Out of inaccurate records on the computer, data on paper make up 80%. Out of inaccurate data on paper, records on the computer account for 48%. Out of all data, inaccurate data, both on the computer and on paper, make up 12%. What is the percent of accurate data among all data, either on the computer or on paper?

A. 45% B. 56% C. 72% D. 86% E. 90%

4. (Integer) If *n* is greater than 20, what number is closest to $n^{100} - n^{90}$?

A. n^{10} B. n^{90} C. n^{99} D. n^{100} E. n^{19}

5. (Inequality) Which of x, x^2, and $\frac{1}{x}$ is the greatest?

(1) $x > 0$.

(2) $x < 1$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

6. (Speed Rate) Tom drove from *X* to *Y* and then returned from *Y* to *X*. He took different routes back and forth. He traveled at 40 miles per hour from *X* to *Y* and traveled 50 miles per hour from *Y* to *X*. What was his average speed for the entire trip?

(1) The distance of the return trip was $\frac{4}{5}$ of the entire distance.

(2) The distance of the return trip was 160 miles.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

7. (Set) In school *V*, all students must take at least one of Mathematics, Physics, or Biology. If none of the students took all 3 classes, the number of students who took Mathematics is 18, the number of students who took Physics is 23, and the number of students who took Biology is 20.
Also, 3 students took both Mathematics and Physics, 8 students took both Physics and Biology, and 6 students took both Biology and Mathematics. What is the total number of students in school *V*?

A. 11 B. 22 C. 33 D. 44 E. 55

8. (Integer) *x* and *y* are positive integers. When *x* is divided by *y*, the remainder is 6 and $\frac{x}{y}$ = 6.12. What is the value of *x*?

A. 6 B. 50 C. 206 D. 306 E. 336

9. (Sequence) A sequence $Z_n = 2n - 1$ for a positive integer *n*. Let S_n be the sum of Z_m where *m* is an integer from 1 to *n*. Which of the following can be the value of S_n?

A. 12 B. 24 C. 36 D. 48 E. 60

10. (Integer) If *n* is a positive integer, how many factors does *n* have?

(1) $\frac{n}{5}$ is a prime number.

(2) *n* has only two different prime factors.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

11. (Sequence) John deposits $10,000 in an account with an annual interest rate of *x* percent (compound quarterly). If no other activity takes place in this account, is John's account balance over $10,500 after 1 year?

(1) $(1 + \frac{x}{200})^2 > 1.05$.

(2) $(1 + \frac{x}{400})^2 > 1.05$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

12. (Sequence) There is a sequence A_n for a positive integer n such that when A_n is divided by A_{n-1}, the remainder is A_{n-2}. If $A_2 = 7$ and $A_1 = 0$, which of the following can be the value of A_4?

A. 48 B. 50 C. 52 D. 56 E. 58

13. (Integer) What is the remainder when 10^8 is divided by 11?

A. 0 B. 1 C. 2 D. 3 E. 4

14. (Function) It is well known that the relationship between the revenue from sales of products and the total cost of labor and materials is represented by the function, $R(c) = -10c^2 + kc + m$, where $R(c)$ is the revenue and c is the total cost. When the revenue from sales of products is at its greatest, what is the value of the total cost?

(1) $k = 200$.

(2) $m = 2{,}000$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

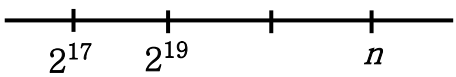
15. (Integer) The remainder of *n* is 1 when divided by 15, 35, and 125. What is the smallest possible integer value of *n*?

A. 1,926 B. 2,126 C. 2,326 D. 2,426 E. 2,626

16. (Ratio) Each of 3 cups has the same *n* number of marbles. When marbles from the 1st cup are moved to the other 2 cups, the ratio of marbles in each cup is 2 : 4 : 3. Then, which of the following represents the number of marbles that are moved from the 1st cup, in terms of *n*?

A. $\frac{n}{4}$ **B.** $\frac{n}{3}$ **C.** $\frac{n}{2}$ **D.** n **E.** $\frac{4n}{5}$

17. (Exponent)



If the distances between two consecutive points are the same as shown above, what is the value of *n*?

A. $3(2^{18})$ B. $3(2^{19})$ C. $5(2^{18})$ D. $5(2^{19})$ E. $7(2^{18})$

18. (Integer) If *n* is the remainder when 123,456,789 is divided by 4 and *m* is the remainder when 987,654,321 is divided by 8, what is the sum of *n* and *m*?

A. 2 B. 3 C. 4 D. 5 E. 6

19. (Exponent) What is the value of $\frac{x^{n^2-n+2}}{x^{(n-2)(n+1)}}$?

(1) $n = 5$.

(2) $x = 2$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

20. (Sequence) There is a sequence A_n and n is a positive integer such that $A_1 = a$, $A_2 = b$, and $A_{n+2} = A_{n+1}A_n$. Is $A_6 < 0$?

(1) $a < 0$.

(2) $ab < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

21. (Integer) On the 25th of November in 2020, John was trying to make a new plan for January 2021. However, he did not have a new calendar that included January 2021. Then, in order to match a new plan for January 2021, which of the following months of 2020 should he look at?

A. March B. April C. May D. June E. July

22. (Probability) ***n*** **is an integer from 21 to 30. What is the probability that** $n(n^2-1)$ **is divisible by 6?**

A. 0.3 **B. 0.42** **C. 0.48** **D. 0.72** **E. 1.00**

23. (Probability) Factory ***X*** **and factory** ***Y*** **are among the 5 factories in a certain manufacturing business. If the CEO must visit the 5 factories every day, in how many different possible orders can the CEO go to the factories, so that factory** ***X*** **is ahead of factory** ***Y*****?**

A. 24 **B. 30** **C. 36** **D. 48** **E. 60**

24. (Statistics) We define the harmonic mean as the reciprocal of the average (arithmetic mean) of the reciprocals of a set of specified numbers. What is the harmonic mean of 2, 3, and 6?

A. $\frac{1}{3}$ **B.** $\frac{1}{2}$ **C. 2** **D. 3** **E. 4**

25. (Integer) If ***a*** **and** ***b*** **are integers, and** $(a-b)^2+8b^2=108$**, how many ordered pairs are possible for** (a, b)**?**

A. 2 **B. 4** **C. 6** **D. 8** **E. 10**

26. (Statistics) In an exam, 5 people have different scores ranging from 0 to 100, inclusive. The scores are all integers. If the median score is 70, what is the greatest possible average (arithmetic mean) score?

A. 80.2 B. 81.2 C. 82.2 D. 83.2 E. 84.4

27. (Integer) If $\{x\}$ is the greatest integer less than or equal to x, what is the value of $\{\frac{1}{2}\} + \{\frac{7}{5}\} + \{\frac{8}{3}\}$?

A. 1 B. 2 C. 3 D. 4 E. 5

28. (Integer) If x and y are positive integers, what is the value of x^y?

(1) $y^x = 9$.

(2) $x^{2y} = 64$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

29. (Absolute Value) If $ab - 2b = (4 - a)b$, what is the value of b?

(1) $|a^2 - 9| \leq 0$.

(2) $a < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

30. (Absolute Value) Is $a < b$?

(1) $|a - b| < |b|$.

(2) $a < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

31. (Statistics) The monthly precipitation in the first 5 months of 1999 was 8, 6, 9, 3, and 5, respectively, in inches. If the sixth month's precipitation in 1999 was either 2, 6, or 11, in inches, for which of these values does the monthly average (arithmetic mean) precipitation for the first 6 months equal the median?

I. 2 II. 6 III. 11

A. I only B. II only C. III only D. I and III only E. I, II, and III

[Answer Key]

1	A	**2**	E	**3**	C	**4**	D
5	C	**6**	A	**7**	D	**8**	D
9	C	**10**	C	**11**	D	**12**	D
13	B	**14**	A	**15**	E	**16**	B
17	C	**18**	A	**19**	B	**20**	B
21	C	**22**	E	**23**	E	**24**	D
25	D	**26**	B	**27**	C	**28**	B
29	B	**30**	E	**31**	D		

[Solution]

1. (Integer) If *a*, *b*, and *c* are integers, 33 = *abc*, and *a* < *b* < *c*, what is the smallest possible value of *a*?

A. -33 B. -3 C. 3 D. 11 E. 33

(Solution)

Since we must consider negative numbers and find the 'hidden 1', we have 33 = (-33)·(-1)·(1). Thus, the smallest possible value of *a* is -33.

Therefore, A is the correct answer.

Answer: A

2. (Inequality) If *a* is an integer greater than 4, but less than 21 and *b* is an integer greater than 14 but less than 31, what is the range of $\frac{a}{b}$?

A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{5}{6}$ D. 1 E. $\frac{7}{6}$

(Solution)

Since *a* and *b* are integers, 4 < *a* < 21 becomes 5 ≤ *a* ≤ 20, and 14 < *b* < 31 becomes 15 ≤ *b* ≤ 30. Then, from there, we get $\frac{a}{b}$: $\frac{5}{15}$, $\frac{20}{15}$, $\frac{5}{30}$, and $\frac{20}{30}$. Since minimum ≤ $\frac{a}{b}$ ≤ maximum, we get $\frac{1}{6} = \frac{5}{30} \leq \frac{a}{b} \leq \frac{20}{15} = \frac{4}{3}$. Hence, the range = maximum – minimum = $\frac{4}{3} - \frac{1}{6} = \frac{8}{6} - \frac{1}{6} = \frac{7}{6}$.

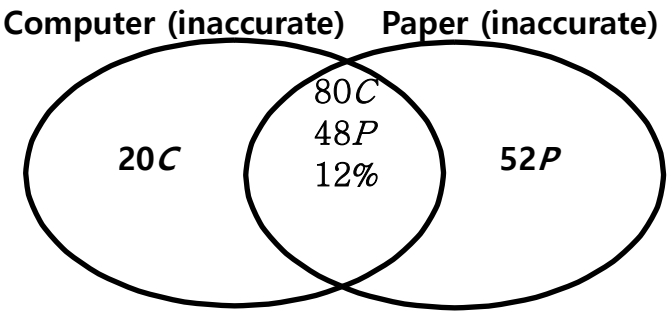
Therefore, E is the correct answer.

Answer: E

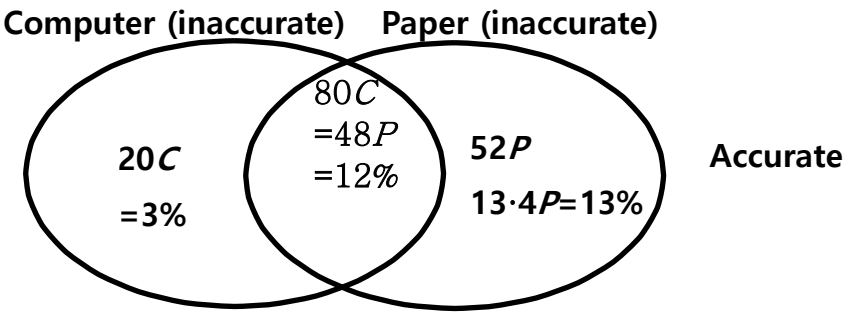
3. (Ratio) All data in a certain school is recorded either on the computer or on paper. Out of inaccurate records on the computer, data on paper make up 80%. Out of inaccurate data on paper, records on the computer account for 48%. Out of all data, inaccurate data, both on the computer and on paper, make up 12%. What is the percent of accurate data among all data, either on the computer or on paper?

A. 45% B. 56% C. 72% D. 86% E. 90%

(Solution)



From the above diagram, we can see that 80*C* = 48*P* = 12% and dividing everything by 4 gives us 20*C* = 12*P* = 3%, 12*P* = 3%, and 4*P* = 1%. Looking back at the diagram:



Thus, inaccurate data = 3% + 12% + 13% = 28%. Then, accurate data = 100% - 28% = 72%.

Therefore, C is the correct answer.

Answer: C

4. (Integer) If n is greater than 20, what number is closest to $n^{100} - n^{90}$?

A. n^{10} B. n^{90} C. n^{99} D. n^{100} E. n^{190}

(Solution)

Working through the given equation, we have $n^{100} - n^{90} = n^{90}(n^{10} - 1) \fallingdotseq n^{90}(n^{10})$ (because subtracting 1 will not substantially change the answer, we can ignore it). Then, we have $n^{90}(n^{10}) = n^{90+10} = n^{100}$.

Therefore, D is the correct answer.

Answer: D

5. (Inequality) Which of x, x^2, and $\frac{1}{x}$ is the greatest?

(1) $x > 0$.

(2) $x < 1$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

Let's look at both conditions together.

From conditions (1) and (2), we have that if $x > 1$, then $\frac{1}{x} < 1 < \sqrt{x} < x < x^2 < x^3$ and if $0 < x < 1$, then $x^3 < x^2 < x < \sqrt{x} < 1 < \frac{1}{x}$. Since the number of answers should be unique for the condition to be sufficient, we can only have one answer for x, x^2, and $\frac{1}{x}$. When $0 < x < 1$, $\frac{1}{x}$ among x, x^2, and $\frac{1}{x}$ is always the greatest. The answer is unique, so both conditions (1) and (2) combined are sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Both conditions (1) and (2) together are sufficient.

Therefore, C is the correct answer.

Answer: C

6. (Speed Rate) Tom drove from *X* to *Y* and then returned from *Y* to *X*. He took different routes back and forth. He traveled at 40 miles per hour from *X* to *Y* and traveled 50 miles per hour from *Y* to *X*. What was his average speed for the entire trip?

(1) The distance of the return trip was $\frac{4}{5}$ of the entire distance.

(2) The distance of the return trip was 160 miles.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

Let d_1 and t_1 be the distance from *X* to *Y* and the time it took Tom to drive from *X* to *Y*, respectively. Let d_2 and t_2 be the distance from *Y* to *X* and the time it took Tom to drive from *Y* to *X*, respectively. So, we get $40t_1 = d_1$, $50t_2 = d_2$ or $t_1 = \frac{d_1}{40}$ and $t_2 = \frac{d_2}{50}$. We have to find the average speed for his entire trip, which equals:

$$\text{Average speed} = \frac{Total\ distance}{Total\ time} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{d_1 + d_2}{\frac{d_1}{40} + \frac{d_2}{50}}$$

So, if we know the relationship between d_1 and d_2, we can find the average speed.

Let's look at each condition separately.

Condition (1) tells us that the distance of the return trip was $\frac{4}{5}$ of the entire distance, so we'll let the entire distance be $5d$ miles. Then the distance coming back is $5d(\frac{4}{5}) = 4d$ miles, which means that $d_2 = 4d$ miles.

The distance going to the destination is $5d - 4d = d$ miles, which means that $d_1 = d$ miles. Substituting $d_1 = d$ and $d_2 = 4d$ gives us

$$\text{Average speed} = \frac{d_1 + d_2}{\frac{d_1}{40} + \frac{d_2}{50}} = \frac{d + 4d}{\frac{d}{40} + \frac{4d}{50}} = \frac{200(d + 4d)}{200(\frac{d}{40} + \frac{4d}{50})} = \frac{200(5d)}{5d + 16d} = \frac{1{,}000d}{21d} = \frac{1{,}000}{21}$$

The answer is unique, so the condition is sufficient according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (2) tells us that the distance of the return trip was 160 miles. However, this does not give us enough information to determine the relationship between d_1 and d_2. The answer is not unique, so the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (1) alone is sufficient.

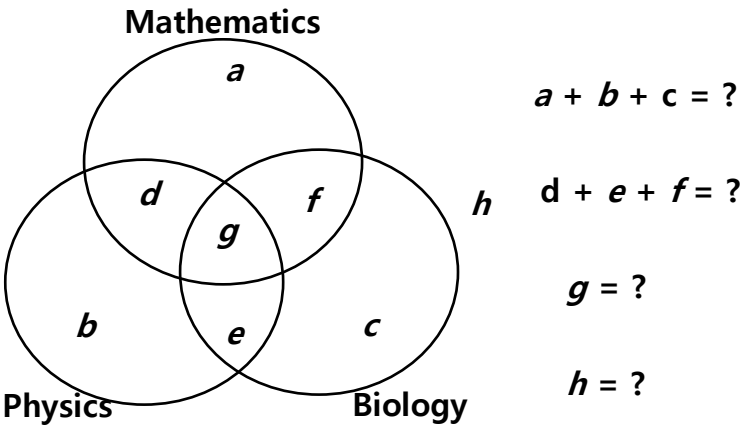
(Note) When the question asks for the average speed (ratio), then the condition with a ratio wins over the condition with a number. So, condition (1) with $\frac{4}{5}$ is sufficient rather than condition (2) with 160.

Therefore, A is the correct answer.
Answer: A

7. (Set) In school *V*, all students must take at least one of Mathematics, Physics, or Biology. If none of the students took all 3 classes, the number of students who took Mathematics is 18, the number of students who took Physics is 23, and the number of students who took Biology is 20.
Also, 3 students took both Mathematics and Physics, 8 students took both Physics and Biology, and 6 students took both Biology and Mathematics. What is the total number of students in school *V*?

A. 11 B. 22 C. 33 D. 44 E. 55

(Solution)



Students who take Mathematics = $a + d + g + f = 18$.
Students who take Physics = $b + e + g + d = 23$.

Students who take Biology = $c + f + g + e = 20$.
If we add the three equations above, we get $a + d + g + f + b + e + g + d + c + f + g + e = 18 + 23 + 20$, or $(a + b + c) + 2(d + e + f) + 3g = 61$.

Students who take both Mathematics and Physics = $d + g = 3$.
Students who take both Physics and Biology = $e + g = 8$.
Students who take both Biology and Mathematics = $f + g = 6$.

If we add the three equations above, we get $d + g + e + g + f + g = 3 + 8 + 6$, or $(d + e + f) + 3g = 17$.

Also, $h = 0$, and $g = 0$. If we substitute 17 for $d + e + f$ into the first equation, we get $(a + b + c) + 2(d + e + f) + 3g = 61$, $(a + b + c) + 2 \cdot 17 + 3 \cdot 0 = 61$, $a + b + c + 34 = 61$, and $a + b + c = 27$.
The total number of students = $(a + b + c) + (d + e + f) + g + h = 27 + 17 + 0 + 0 = 44$.

Therefore, D is the correct answer.
Answer: D

8. (Integer) x and y are positive integers. When x is divided by y, the remainder is 6 and $\frac{x}{y} = 6.12$. What is the value of x?

A. 6 B. 50 C. 206 D. 306 E. 336

(Solution)
Since, when x is divided by y, the remainder is 6, we get $x = yQ + 6$, where Q = any integer and $y > 6$. If we divide both sides of $y > 6$ by y, we get $1 > \frac{6}{y}$, which is a decimal. If we substitute $x = yQ + 6$ into $\frac{x}{y}$, we get $\frac{x}{y} = \frac{yQ + 6}{y} = Q$

+ $\frac{6}{y}$ = 6.12 = 6 + 0.12. So, we get Q = 6 and $\frac{6}{y}$ = 0.12, 6 = 0.12y, or $y = \frac{6}{0.12}$ = 50.

Then, $x = yQ + 6 = 50 \cdot 6 + 6 = 306$.

Therefore, D is the correct answer.

Answer: D

9. (Sequence) A sequence $Z_n = 2n - 1$ for a positive integer n. Let S_n be the sum of Z_m where m is an integer from 1 to n. Which of the following can be the value of S_n?

A. 12 B. 24 C. 36 D. 48 E. 60

(Solution)

Substituting n = 1, 2, 3,… into S_n gives us:

$S_1 = Z_1 = 1 = 1^2$.

$S_2 = Z_1 + Z_2 = 1 + 3 = 4 = 2^2$.

$S_3 = Z_1 + Z_2 + Z_3 = 1 + 3 + 5 = 9 = 3^2$.

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So, we get $S_n = n^2$. Then, the option with a square number can be the answer, which is option C, since $36 = 6^2$.

Therefore, C is the correct answer.

Answer: C

10. (Integer) If *n* is a positive integer, how many factors does *n* have?

(1) $\frac{n}{5}$ is a prime number.

(2) *n* has only two different prime factors.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

We have to find the number of the factors of *n* if *n* is a positive integer.

Follow the second and the third step: From the original condition, we have 1 variable (*n*), and 0 equations. To match the number of variables with the number of equations, we need 1 equation. Since conditions (1) and (2) will provide 1 equation each, D would most likely be the answer.

Let's look at each condition separately.

Condition (1) tells us that $\frac{n}{5}$ is a prime number, from which we cannot determine the unique number of the factors of *n*. For example, if $\frac{n}{5}$ = 3, *n* = $3^1 \cdot 5^1$, then the number of factors of *n* is (1 + 1)(1 + 1) = 2·2 = 4. However, if $\frac{n}{5}$ = 5, *n* = 5^2, then the number of factors of *n* is 2 + 1 = 3. The answer is not unique, so the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (2) tells us that n has only two different prime factors, from which we cannot determine the unique number of the factors of n. For example, if $n = 3^1 \cdot 5^1$, the number of factors of n is $(1 + 1)(1 + 1) = 2 \cdot 2 = 4$. However, if $n = 3^3 \cdot 5^1$, the number of factors of n is $(3 + 1)(1 + 1) = 4 \cdot 2 = 8$. The answer is not unique, and the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Both conditions (1) and (2) combined give us that n should be $3^1 \cdot 5^1$ and the number of factors of n is $(1 + 1)(1 + 1) = 2 \cdot 2 = 4$. So, the answer is unique, and the conditions together are sufficient according to Common Mistake Type 2, which states that the number of answers must be only one.

Both conditions (1) and (2) together are sufficient.

Therefore, C is the correct answer.

Answer: C

11. (Sequence) John deposits $10,000 in an account with an annual interest rate of x percent (compound quarterly). If no other activity takes place in this account, is John's account balance over $10,500 after 1 year?

(1) $(1 + \frac{x}{200})^2 > 1.05$.

(2) $(1 + \frac{x}{400})^2 > 1.05$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

We have to verify whether John's account balance is over $10,500 after 1 year, which is equal to asking whether $10{,}000(1 + \frac{x}{400})^4 > 10{,}500$ or whether $(1 + \frac{x}{400})^4 > \frac{10{,}500}{10{,}000} = 1.05$.

Follow the second and the third step: From the original condition, we have 1 variable (x). To match the number of variables with the number of equations, we need 1 equation. Since conditions (1) and (2) will provide 1 equation each, D would most likely be the answer.

Thus, let's look at condition (2). It tells us that $(1 + \frac{x}{400})^2 > 1.05$. If we square both sides of the inequality, we get $(1 + \frac{x}{400})^4 = ((1+\frac{x}{400})^2)^2 > 1.05^2 > 1.05$, so we get yes as an answer. The answer is unique, yes, and the answer is sufficient according to Common Mistake Type 1, which states that the answer must be a unique yes or no.

Here, we can apply Common Mistake Type 4(B), which states that if you get the answer A and B too easily, consider answer choice D.

Condition (1) tells us that $(1 + \frac{x}{200})^2 > 1.05$, from which it follows that $(1 + \frac{x}{400})^4 \geq (1 + \frac{x}{200})^2 > 1.05$ or $(1 + \frac{x}{400})^4 > 1.05$ because the total amount, which includes the quarterly interest rate, is larger than semi-annually, so we get yes as an answer. The answer is unique, yes, and the answer is sufficient according to Common Mistake Type 1, which states that the answer must be a unique yes or no.

We also apply Tip 1, which states that if the value of condition (1) is equal to the value of condition (2), the most likely answer is D.

Each condition alone is sufficient.

Therefore, D is the correct answer.

Answer: D

12. (Sequence) There is a sequence A_n for a positive integer n such that when A_n is divided by A_{n-1}, the remainder is A_{n-2}. If $A_2 = 7$ and $A_1 = 0$, which of the following can be the value of A_4?

A. 48 B. 50 C. 52 D. 56 E. 58

(Solution)

If we substitute $n = 3$ into the sequence, we get $A_3 = A_2Q + A_1 = 7Q + 0 = 7Q$, where Q is any integer. If we substitute $n = 4$ into the sequence, we get $A_4 = A_3P + A_2 = 7QP + 7 = 7(QP + 1)$ = a multiple of 7, where P is any integer. Thus, the option with a multiple of 7 can be the answer.

Therefore, D is the correct answer, since $56 = 7 \cdot 8$.

Answer: D

13. (Integer) What is the remainder when 10^8 is divided by 11?

A. 0 B. 1 C. 2 D. 3 E. 4

(Solution)

Since dividing after computing is equal to computing after dividing, dividing by 11 is the same as treating 10 as -1 since $10 = 11 \cdot 0 + 10 = 11 \cdot 1 - 1$. Then, the remainder when 10^8 is divided by 11 is the same as when $(-1)^8$ is divided by 11, and $(-1)^8 = 1$.

Therefore, B is the correct answer.

Answer: B

14. (Function) It is well known that the relationship between the revenue from sales of products and the total cost of labor and materials is represented by the function, $R(c) = -10c^2 + kc + m$, where $R(c)$ is the revenue and c is the total cost. When the revenue from sales of products is at its greatest, what is the value of the total cost?

(1) $k = 200$.

(2) $m = 2{,}000$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

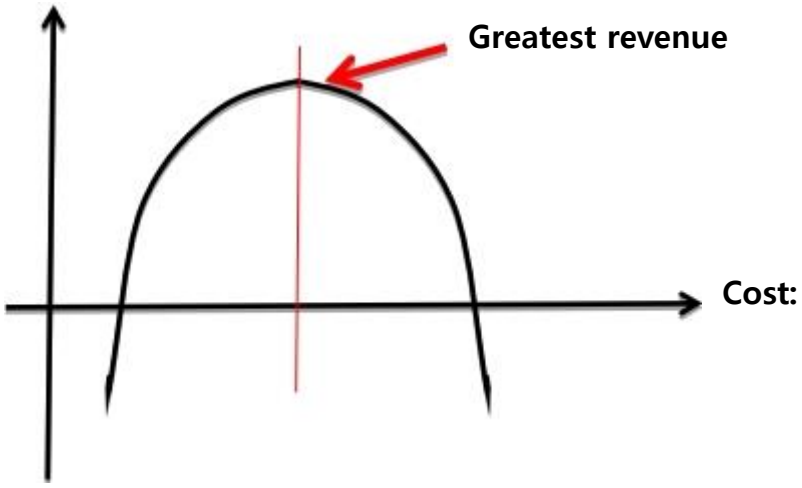
(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

Revenue $R(c) = -10c^2 + kc + m$



Generally, $y = f(x) = ax^2 + bx + c$ has its maximum when $x = -\frac{b}{2a}$.

Since the revenue from the sales of products is at its maximum, from $R(c) = -10c^2 + kc + m$, we get the highest revenue when $c = -\frac{k}{2\cdot(-10)} = \frac{k}{20}$. So, we only have to find the value of k.

Thus, let's look at condition (1). It tells us that $k = 200$, which is exactly what we are looking for. The answer is unique, so condition (1) is sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (2) tells us that $m = 2{,}000$, from which we cannot determine the unique value of k. The answer is not unique, so the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (1) alone is sufficient.

Therefore, A is the correct answer.

Answer: A

15. (Integer) The remainder of n is 1 when divided by 15, 35, and 125. What is the smallest possible integer value of n?

A. 1,926 B. 2,126 C. 2,326 D. 2,426 E. 2,626

(Solution)

Let's say $N = n - 1$. Then, since N is divisible by 15, 35, and 125, these numbers become the factors of N. Also, this means N is a multiple of 15, 35, and 125. In other words, since N is a multiple of the three integers, it is a common multiple. Since the question asks what is the smallest possible integer value of n, we have to find the least common multiple of N.

Since the least common multiple is the greatest possible value of exponents, from $15 = 3\cdot5$, $35 = 5\cdot7$, and $125 = 5^3$, we get $3\cdot5^3\cdot7^1 = 2{,}625$ as their least common multiple. This means 2,625 is divisible by 15, 35 and 125, and $N =$

2,625. Since $N = n - 1$, $n = N + 1$, and $n = 2{,}625 + 1 = 2{,}626$.

Therefore, E is the correct answer.

Answer: E

16. (Ratio) Each of 3 cups has the same n number of marbles. When marbles from the 1st cup are moved to the other 2 cups, the ratio of marbles in each cup is 2 : 4 : 3. Then, which of the following represents the number of marbles that are moved from the 1st cup, in terms of n?

A. $\frac{n}{4}$ B. $\frac{n}{3}$ C. $\frac{n}{2}$ D. n E. $\frac{4n}{5}$

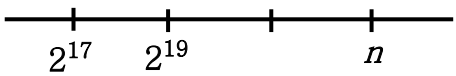
(Solution)

Since $n : n : n \dashrightarrow 2k : 4k : 3k$, the total number of marbles does not change. Then, from $n + n + n = 3n = 2k + 4k + 3k = 9k$, we get $3n = 9k$, and $n = 3k$. In other words, from $3k : 3k : 3k \dashrightarrow 2k : 4k : 3k$. So, since from the 1st cup, $3k \dashrightarrow 2k$, from the 2nd cup, $3k \dashrightarrow 4k$, and from the 3rd cup, $3k \dashrightarrow 3k$. the total number of marbles moved is $3k - 2k = k$, since $3k$ from the 1st cup moved to $4k$ in the 2nd cup. Thus, $k = \frac{n}{3}$.

Therefore, B is the correct answer.

Answer: B

17. (Exponent)

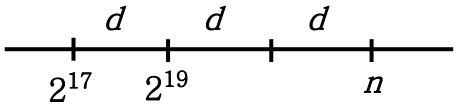


If the distances between two consecutive points are the same as shown above, what is the value of *n*?

A. $3(2^{18})$ B. $3(2^{19})$ C. $5(2^{18})$ D. $5(2^{19})$ E. $7(2^{18})$

(Solution)

If we let the distance between two points be *d*, we get:



$d = 2^{19} - 2^{17} = 2^{17}(2^2-1) = 3(2^{17})$.

$n = 2^{19} + 2d = (2^2)(2^{17})+2\cdot3(2^{17}) = 4(2^{17}) + 6(2^{17}) = (4 + 6)(2^{17}) = 10(2^{17}) = 5\cdot2(2^{17}) = 5(2^{18})$.

Therefore, C is the correct answer.

Answer: C

18. (Integer) If *n* is the remainder when 123,456,789 is divided by 4 and *m* is the remainder when 987,654,321 is divided by 8, what is the sum of *n* and *m*?

A. 2 B. 3 C. 4 D. 5 E. 6

(Solution)

We have to know that the remainder when an integer *a* is divided by 4 is same as the tens and the units digits of *a* divided by 4 and the remainder when an integer *b* is divided by 8 is same as the hundreds, the tens, and the units digits of *b* divided by 8.

In other words, *n* is the remainder when 89 divided 4, and *m* is the remainder

when 321 is divided by 8. Since 89 = 4·22 + 1, we get n = 1 and since 321 = 8·40 + 1, we get m = 1. Thus, $n + m = 1 + 1 = 2$.

Therefore, A is the correct answer.

Answer: A

19. (Exponent) What is the value of $\frac{x^{n^2-n+2}}{x^{(n-2)(n+1)}}$?

(1) $n = 5$.

(2) $x = 2$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

We have to find the value of $\frac{x^{n^2-n+2}}{x^{(n-2)(n+1)}}$, which is equal to $x^{n^2-n+2-(n-2)(n+3)}$ = $x^{n^2-n+2-(n^2-n-2)}$ = $x^{n^2-n+2-n^2+n+2}$ = x^4, so we have to determine the value of x.

Thus, let's look at condition (2). It tells us that x = 2, which is exactly what we are looking for. The answer is unique, so the condition is sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (1) tells us that n = 5, from which we cannot determine the unique

value of x. The answer is not unique, so the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.
Condition (2) alone is sufficient.

Therefore, B is the correct answer.
Answer: B

20. (Sequence) There is a sequence A_n and n is a positive integer such that $A_1 = a$, $A_2 = b$, and $A_{n+2} = A_{n+1}A_n$. Is $A_6 < 0$?
(1) $a < 0$.
(2) $ab < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)
Now we will solve this DS question using the Variable Approach.
Let's apply the 3 steps suggested previously.
Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.
If we substitute $n = 1$ into $A_{n+2} = A_{n+1}A_n$, we get $A_3 = A_2A_1 = ba$. If we substitute $n = 2$ into $A_{n+2} = A_{n+1}A_n$, we get $A_4 = A_3A_2 = (ba)b = b^2a$. If we substitute $n = 3$ into $A_{n+2} = A_{n+1}A_n$, we get $A_5 = A_4A_3 = (b^2a)(ba) = b^3a^2$. If we substitute $n = 4$ into $A_{n+2} = A_{n+1}A_n$, we get $A_6 = A_5A_4 = (b^3a^2)(b^2a) = b^5a^3$.
We have to verify whether $A_6 < 0$, which is equal to asking whether $b^5a^3 < 0$ or whether $ba < 0$. Since b^4a^2 is positive, the sign of inequality does not change

if we divide both sides of $b^5a^3 < 0$ by b^4a^2. So, we only need to verify whether $ba < 0$.

Thus, let's look at condition (2). It tells us that $ab < 0$, which is exactly what we are looking for, and we get yes as an answer. The answer is unique, yes, so the condition is sufficient according to Common Mistake Type 1, which states that the answer must be a unique yes or no.

Condition (1) tells us that $a < 0$, from which we cannot determine whether $ba < 0$. For example, if $a = -1$ and $b = 1$, then $ba = 1\cdot(-1) = -1 < 0$, and we get yes as an answer. However, if $a = -1$ and $b = -1$, then $ba = (-1)\cdot(-1) = 1 > 0$, and we get no as an answer. The answer is not unique, yes and no, so the condition is not sufficient, according to Common Mistake Type 1, which states that if we get both yes and no as an answer, it is not sufficient.

Condition (2) alone is sufficient.

Therefore, B is the correct answer.

Answer: B

21. (Integer) On the 25th of November in 2020, John was trying to make a new plan for January 2021. However, he did not have a new calendar that included January 2021. Then, in order to match a new plan for January 2021, which of the following months of 2020 should he look at?

A. March B. April C. May D. June E. July

(Solution)

There are 7 days in 1 week. If the number is divided by 7 and the remainder is the same, their days will be the same as well.

In other words, since May has 31 days, if we divide it by 7, we get a remainder of 3, from $31 = 7\cdot4 + 3$. Since June has 30 days, if we divide it by 7, we get a remainder of 2, from $30 = 7\cdot4 + 2$. The remainder for July is 3, of August is 3, for September is 2, for October is 3, for November is 2 and for December is 3.

If we add up all the remainders, we get 3 + 2 + 3 + 3 + 2 + 3 + 2 + 3 = 21. Since 21 = 7·3 + 0, the remainder is 0. Thus, May of this year has the same calendar as January of next year. So, in order to make a plan for January 2021, he should look at May of 2020.

Therefore, C is the correct answer.
Answer: C

22. (Probability) n is an integer from 21 to 30. What is the probability that $n(n^2-1)$ is divisible by 6?

A. 0.3 B. 0.42 C. 0.48 D. 0.72 E. 1.00

(Solution)

$$\text{Probability} = \frac{\text{Number of one particular case}}{\text{Total number of cases}}.$$

Since n is an integer from 21 to 30, the total number of cases is also 10. $n(n^2-1) = (n-1)\cdot n\cdot(n+1)$, which is equal to the multiple of 3 consecutive terms, and it is always a multiple of 6. Since $n(n^2-1)$ is always divisible by 6, the number of $n(n^2-1)$ that are divisible by 6 is also 10. So, the probability that $n(n^2-1)$ is divisible by 6 is $\frac{10}{10} = 1$.

Therefore, E is the correct answer.
Answer: E

23. (Probability) Factory *X* and factory *Y* are among the 5 factories in a certain manufacturing business. If the CEO must visit the 5 factories every day, in how many different possible orders can the CEO go to the factories, so that factory *X* is ahead of factory *Y*?

A. 24 B. 30 C. 36 D. 48 E. 60

(Solution)

In the case of permutations that include the same letters, we divide it by their factorials. For example, if we line up *c*, *o*, *f*, *f*, *e*, and *e*, there are a total of 6 letters, and we get 6!. Since *f* and *e* appear twice each, we can divide 6! by 2! and 2!. So, the total number of possibilities of lining up c, o, f, f, e, and e is $\frac{6!}{2!2!}$.

Next, in the case of permutations, if letters have a certain order, we have to treat them as the same case. In other words, let the CEO visit *X*, *Y*, *A*, *B*, and *C*, then we should treat *X* = *Y* since *X* is ahead of *Y*. So, we get *X*, *X*, *A*, *B*, *C*. The different possible orders become $\frac{5!}{2!}$ (since we have 2 *X*'s). Thus, $\frac{5!}{2!}$ = $\frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1}$ = 5·4·3 = 60.

Therefore, E is the correct answer.

Answer: E

24. (Statistics) We define the harmonic mean as the reciprocal of the average (arithmetic mean) of the reciprocals of a set of specified numbers. What is the harmonic mean of 2, 3, and 6?

A. $\frac{1}{3}$ **B.** $\frac{1}{2}$ **C. 2** **D. 3** **E. 4**

(Solution)

We have to calculate H(2, 3, 6), which means the harmonic mean of 2, 3, and 6.

1st step: The reciprocals of a set of specified numbers,

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$$

2nd step: The average (arithmetic mean),

$$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}{3} = \frac{\frac{3+2+1}{6}}{3} = \frac{1}{3}.$$

3rd step: The reciprocal of the average,

$$\frac{1}{\frac{1}{3}} = 3.$$

Thus, H(2, 3, 6) = 3.

Therefore, D is the correct answer.

Answer: D

25. (Integer) If *a* and *b* are integers, and $(a - b)^2 + 8b^2 = 108$, how many ordered pairs are possible for (*a*, *b*)?

A. 2 **B. 4** **C. 6** **D. 8** **E. 10**

(Solution)

For $(a - b)^2 + 8b^2 = 108$, there are 2 cases such that $(a - b)^2 = 100$ and $b^2 = 1$ or $(a - b)^2 = 36$ and $b^2 = 9$. Thus, from $(a - b)^2 = 100$ and $b^2 = 1$, we get $a - b = \pm 10$ and

b = ±1, and from $(a - b)^2 = 36$ and $b^2 = 9$, we get $a – b$ = ±6 and b = ±3.
When $a – b$ = 10 and b = 1, we get $a – 1 = 10$ or a = 11, so $(a, b) = (11, 1)$.
When $a – b$ = -10 and b = 1, we get $a – 1 = -10$ or a = -9, so $(a, b) = (-9, 1)$.
When $a – b$ = 10 and b = -1, we get $a - (-1) = 10$ or a = 9, so $(a, b) = (9, -1)$.
When $a – b$ = -10 and b = -1, we get $a - (-1) = -10$ or a = -11, so $(a, b) = (-11, -1)$.
When $a – b$ = 6 and b = 3, we get $a – 3 = 6$ or a = 9, so $(a, b) = (9, 3)$.
When $a – b$ = -6 and b = 3, we get $a – 3 = -6$ or a = -3, so $(a, b) = (-3, 3)$.
When $a – b$ = 6 and b = -3, we get $a - (-3) = 6$ or a = 3, so $(a, b) = (3, -3)$.
When $a – b$ = -6 and b = -3, we get $a - (-3) = -6$ or a = -9, so $(a, b) = (-9, -3)$.
This means we have 8 pairs.

Therefore, D is the correct answer.
Answer: D

26. (Statistics) In an exam, 5 people have different scores ranging from 0 to 100, inclusive. The scores are all integers. If the median score is 70, what is the greatest possible average (arithmetic mean) score?

A. 80.2 B. 81.2 C. 82.2 D. 83.2 E. 84.4

(Solution)
To be the greatest possible value of the average score, the sum of the scores is the largest. So, ( ) + ( ) + 70 + ( ) + ( ) should be the greatest and since the scores are different positive integers, we get (68) + (69) + 70 + (99) + (100) = 406.
So, the greatest possible average score is $\frac{406}{5}$ = 81.2.

Therefore, B is the correct answer.
Answer: B

27. (Integer) If {*x*} is the greatest integer less than or equal to *x*, what is the value of $\{\frac{1}{2}\} + \{\frac{7}{5}\} + \{\frac{8}{3}\}$?

A. 1 B. 2 C. 3 D. 4 E. 5

(Solution)

Since {*x*} is the greatest integer less than or equal to *x*, it means that {*x*} rounds down. Thus, we get $\{\frac{1}{2}\} + \{\frac{7}{5}\} + \{\frac{8}{3}\}$ = {0.5} + {1.4} + {2.6666..} = 0 + 1 + 2 = 3.

Therefore, C is the correct answer.

Answer: C

28. (Integer) If *x* and *y* are positive integers, what is the value of x^y?

(1) $y^x = 9$.

(2) $x^{2y} = 64$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

We have to find the value of x^y.

Follow the second and the third step: From the original condition, we have 2 variables (x and y) and no equations. To match the number of variables with the number of equations, we need 2 equations. Since conditions (1) and (2) will provide 1 equation each, C would most likely be the answer.

Let's look at both conditions (1) and (2) together.

They tell us that $y^x = 9$ and $x^{2y} = 64$. Since $y^x = 3^2$ and $x^{2y} = 64 = 2^{2\cdot3}$, we get $x = 2$ and $y = 3$. The answer is unique, so the conditions combined are sufficient. So, C seems to be the answer.

However, since this is an integer question, one of the key questions, we should apply Common Mistake Type 4(A), which states that if we get C too easily as an answer, consider A or B as an answer.

Let's look at each condition separately.

Condition (1) tells us that $y^x = 9$, from which we cannot determine the unique value of x^y. Since we should find the 'hidden 1', we get $9 = 3^2 = 9^1$ and $(x, y) = (2, 3)$ or $(1, 9)$. The answer is not unique, so the condition is not sufficient, according to Common Mistake Type 2, which states that the number of answers must be only one.

Condition (2) tells us that $x^{2y} = 64$, from which we get $(x^y)^2 = x^{2y} = 64 = 8^2$, so $x^y = 8$. The answer is unique, so the condition is sufficient according to Common Mistake Type 2, which states that the number of answers must be only one.

According to the definition of DS questions, when both C and B are answers, B is the correct answer rather than C.

Condition (2) alone is sufficient.

Thus, B is the correct answer.

Answer: B

29. (Absolute Value) If $ab - 2b = (4 - a)b$, what is the value of b?

(1) $|a^2 - 9| \le 0$.

(2) $a < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.

B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.

C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.

D. EACH condition ALONE is sufficient.

E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)

Now we will solve this DS question using the Variable Approach.

Let's apply the 3 steps suggested previously.

Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.

Modifying the original condition gives us $ab - 2b = 4b - ab$, $2ab - 6b = 0$, or $2b(a - 3) = 0$ and we have to find the value of b.

Follow the second and the third step: From the original condition, we have 2 variables (a and b) and 1 equation ($2b(a - 3) = 0$). To match the number of variables with the number of equations, we need 1 more equation. Since conditions (1) and (2) will provide 1 equation each, D would most likely be the answer.

Let's look at each condition separately.

Condition (1) tells us that $|a^2 - 9| \le 0$, from which we get $a^2 - 9 = 0$, $a^2 = 9$, or $a = \pm 3$. If we substitute $a = 3$ into $2b(a - 3) = 0$, we get that b can be any number since $2b(3 - 3) = 2b \cdot 0 = 0$. The answer is not unique, so the condition is not sufficient according to Common Mistake Type 2. which states that the number of answers must be only one.

Condition (2) tells us that $a < 0$, from which we get $a - 3$ is always negative and since $2b(a - 3) = 0$, b must be 0. The answer is unique, so the condition is sufficient according to Common Mistake Type 2, which states that the number

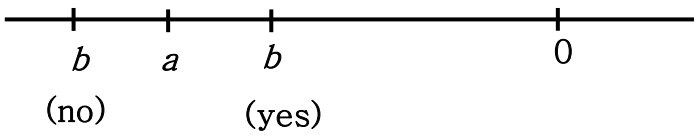
of answers must be only one.
Condition (2) alone is sufficient.

Therefore, B is the correct answer.
Answer: B

30. (Absolute Value) Is $a < b$?
(1) $|a - b| < |b|$.
(2) $a < 0$.

A. Condition (1) ALONE is sufficient, but condition (2) alone is not sufficient.
B. Condition (2) ALONE is sufficient, but condition (1) alone is not sufficient.
C. BOTH conditions TOGETHER are sufficient, but NEITHER condition ALONE is sufficient.
D. EACH condition ALONE is sufficient.
E. Conditions (1) and (2) TOGETHER are NOT sufficient.

(Solution)
Now we will solve this DS question using the Variable Approach.
Let's apply the 3 steps suggested previously.
Follow the first step of the Variable Approach by modifying and rechecking the original condition and the question.
We have to verify whether $a < b$.
Follow the second and the third step: From the original condition, we have 2 variables (a and b) and no equations. To match the number of variables with the number of equations, we need 2 equations. Since conditions (1) and (2) will provide 1 equation each, C would most likely be the answer.
Let's look at both conditions (1) and (2) together. From condition (1), since it is the same as $|a - b| < |b - 0|$, this means that the distance between a and b is smaller than the distance between b and 0. From condition (2), we get $a < 0$.
So, we can draw a number line, as shown below:



It shows that the distance between *a* and *b* is not always smaller than the distance between *b* and 0. The answer is not unique, yes and no, and the conditions combined are not sufficient according to Common Mistake Type 1, which states that if we get both yes and no as an answer, it is not sufficient.
Both conditions (1) and (2) together are not sufficient.

Therefore, E is the correct answer.
Answer: E

31. (Statistics) The monthly precipitation in the first 5 months of 1999 was 8, 6, 9, 3, and 5, respectively, in inches. If the sixth month's precipitation in 1999 was either 2, 6, or 11, in inches, for which of these values does the monthly average (arithmetic mean) precipitation for the first 6 months equal the median?

I. 2 II. 6 III. 11

A. I only B. II only C. III only D. I and III only E. I, II, and III

(Solution)
In order for the median and the mean to equal, it should be consecutive integers or the sum of the numbers should be consistent symmetrically. In other words, we get:

(2), 3, 5, 6, 8, 9
3, 5, 6, (6), 8, 9
3, 5, 6, 8, 9, (11)
(2), 3, 5, 6, 8, 9 --> 2 + 9 = 3 + 8 = 5 + 6 (O)
3, 5, 6, (6), 8, 9 --> 3 + 9 ≠ 5 + 8 (X)

3, 5, 6, 8, 9, 11 --> 3 + 11 = 5 + 9 = 6 + 8 (O)

As shown above, only options I and III work. They have the sum of numbers that are consistent symmetrically, so options I and III have the same average and median.

Therefore, D is the correct answer.

Answer: D

Number of Incorrect Answers	Estimating Score
1~5	49~51
6~8	45~48
9~11	41~44
~12	36~40

